Suggested solution of Assignment 5

(1) We first show that e_1, e_2 form a basis of T_pM where $p=(0,0,0)$.

We consider the Monge patch $X(x,y) = (x, y, x^2+Ky^2)$, then

$X_x = (1, 0, 2x)$, $X_y = (0, 1, 2Ky)$. So at $p=(0,0,0)$,

T_pM is spanned by $\{X_x, X_y\}\big|_{p=(0,0,0)}$ i.e $\{(1,0,0), (0,1,0)\}$.

Next, we find the Matrix of $d\vec{n}_p$ w.r.t e_1, e_2. We see that $d\vec{n}_p = -S_p$ if we consider T_pM as $T_{\vec{n}(p)}S^2$. By direct computation, one can obtain the matrix of S_p under $\{e_1, e_2\}$ is

$$+\begin{pmatrix} 2 & 0 \\ 0 & 2K \end{pmatrix}.$$ Then the matrix of $d\vec{n}_p$ w.r.t $\{e_1,$

Should be

$$-2\begin{pmatrix} 1 & 0 \\ 0 & K \end{pmatrix}.$$

Finally, find the principal curvature of M at p.

For any $v \in T_pM$ with $|v|=1$ ~~$(v=v_1e_1+v_2e_2)$~~ $(v = \cos\theta e_1 + \sin\theta e_2)$, $k_n(v) = \langle S_p(v), v\rangle$

$$= +2(\cos^2\theta + K\sin^2\theta)$$

$$= +2(\cancel{(1-K)\cos}\ K + (1-K)\cos^2\theta)$$

o If $K \cancel{>} < 1$, then $k_{max} = +2$, $k_{min} = +2K$.

o If $0<K \cancel{<} > 1$, then $k_{max} = +2K$, $k_{min} = +2$.

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(2) Please see my tutorial notes (Tutorial 7). 3

(3) Let T, N, B be the Frenet frame along $\alpha(s)$. U = the Gauss map.

Then $\cos\theta = \langle N, U\rangle$ where θ is the angle between U and N.

diff.

$$\Rightarrow \quad -\sin\theta \frac{d\theta}{ds} = \langle N', U\rangle + \langle N, \nabla_{\alpha'} U\rangle \qquad (*)$$

Note that the shape operator of S^2 at p, $S_p = -\frac{1}{R}\, id_{T_pM}$.

and $N' = -\kappa T + \tau B$. Then $(*)$ becomes

$$-\sin\theta \frac{d\theta}{ds} = \tau \langle B, U\rangle .$$

Since B, N, U are tangential to T, B, N, U are on the same plane.

$$\Rightarrow \quad \langle B, U\rangle = \sin\theta \text{ or } -\sin\theta$$

$$\Rightarrow \quad \frac{d\theta}{ds} = \pm\tau$$

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$$\Rightarrow \quad 0 = \int d\theta = \pm \int \tau\, ds .$$

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